Math 1411 Quiz 5 Name KEY

You may use a calculator and your homework, but not your books or notes. There are two (2) problems worth 10 points each. **Show all of your work to receive full/partial credit.**

1) (#25 from 3.4) Find the points of inflection and discuss the concavity of the graph of the function.

$$f(x) = \frac{4}{x^2+1} = 4(x^2+1)^{-1}$$

$$f'(x) = -4(x^2+1)^{-2}(2x) = \frac{-8x}{(x^2+1)^2}$$

$$f''(x) = \frac{-8(x^2+1)^2 - (-8x)[2(x^2+1)(2x)]}{(x^2+1)^4}$$

$$= \frac{\cancel{(x^2+1)}[-8(x^2+1) + 8x(4x)]}{(x^2+1)^{\cancel{4}\,3}} = \frac{-8x^2 - 8 + 32x^2}{(x^2+1)^3}$$

$$= \frac{24x^2 - 8}{(x^2+1)^3} \rightarrow 24x^2 - 8 = 0, \quad x^2 = \frac{1}{3}$$

$$x = \pm\sqrt{1/3} = \pm\frac{\sqrt{3}}{3}$$ ← possible inflection points

	$\left(-\infty, -\frac{\sqrt{3}}{3}\right)$	$\left(-\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}\right)$	$\left(\frac{\sqrt{3}}{3}, \infty\right)$
T.V.	$x = -1$	$x = 0$	$x = 1$
sign of f''	+	−	+
conclusion	C.U.	C.D.	C.U.

concave down on $\left(-\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}\right)$

concave up on $\left(-\infty, -\frac{\sqrt{3}}{3}\right) \cup \left(\frac{\sqrt{3}}{3}, \infty\right)$

Inflection points at $\left(-\frac{\sqrt{3}}{3}, f\left(-\frac{\sqrt{3}}{3}\right)\right) = \left(-\frac{\sqrt{3}}{3}, 3\right)$

and $\left(\frac{\sqrt{3}}{3}, f\left(\frac{\sqrt{3}}{3}\right)\right) = \left(\frac{\sqrt{3}}{3}, 3\right)$

2) (#29 from 3.5) Find the limit.

$$\lim_{x \to -\infty} \frac{2x+1}{\sqrt{x^2 - x}}$$

$$\lim_{x \to -\infty} \frac{(2x+1)\left(\frac{1}{x}\right)}{\sqrt{x^2-x}\,\left(\frac{1}{x}\right)} = \lim_{x \to -\infty} \frac{2 + \frac{1}{x}}{\frac{\sqrt{x^2-x}}{x}} = \lim_{x \to -\infty} \frac{2 + \frac{1}{x}}{\frac{\sqrt{x^2-x}}{-\sqrt{x^2}}} \quad (*)$$

$$= \lim_{x \to -\infty} \frac{2 + \frac{1}{x}}{-\sqrt{\frac{x^2-x}{x^2}}} = \lim_{x \to -\infty} \frac{2 + \underbrace{\frac{1}{x}}_{\to 0}}{-\sqrt{1 - \underbrace{\frac{1}{x}}_{\to 0}}} = \frac{2}{-1} = -2$$

(*) For $x < 0$, $x = -\sqrt{x^2}$